Algorithms for GNSS Positioning in Difficult Scenario

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Satellite navigation is critical in signal-degraded environments such as urban canyons and mountainous area, where many GNSS signals are blocked by natural and artificial obstacles or are strongly degraded. Hence standalone GPS is often unable to guarantee a continuous and accurate positioning. A suitable approach could be the integration of several GNSS. Multi-constellation system guarantees an improved satellite availability with respect to GPS standalone, providing a positioning enhancement in terms of accuracy, continuity and integrity. Currently the ideal candidate for supplement GPS in a multi-constellation approach is the Russian GLONASS. The main purposes of this work are the performance assessment of a GNSS multi-constellation relative to GPS stand-alone and the comparison of Least Squares and Kalman Filter.

KEY WORDS
1. Pseudorange 2. GNSS 3. RAIM 4. Kalman Filter

1. INTRODUCTION. GNSS (Global Navigation Satellite Systems) are worldwide, all-weather navigation systems able to provide tridimensional position velocity and time synchronization to UTC (Coordinated Universal Time) scale (Hoffmann-Wellenhof et al 1992, Kaplan and Hegarty 2006). GNSS positioning is based on the reception of signals transmitted by satellites, hence their performances are related to signal quality and operational scenario. GNSS performances are optimal in open sky with many satellites in view and no degraded signal; in these condition position accuracy in single point positioning is about 10 m (Kaplan and Hegarty 2006). The use of these systems in difficult scenario such as urban canyon and mountainous area is critical, because many GNSS signals are blocked by natural and artificial obstacles or are strongly degraded. Currently GPS (Global Positioning System) is the most widespread GNSS, is a space-based radio-navigation system developed by the US DoD (Department of Defense) and is fully operative since 1994. In critical environments GPS stand-alone is not able to provide accurate and continuous absolute positioning; a possible approach to solving this problem is to consider the combined use of GPS with other GNSS.

GLONASS (Global Navigation Satellite System) is the Russian alter-ego of GPS and since 2003 it is in modernization phase. The GLONASS recent enhancement candidate this system as an alternative to GPS, but also as a component of multi-constellation system.

Another element of a multi-constellation systems will be the European satellite system Galileo that currently has only 4 in orbit satellites. In this research only GPS and GLONASS will be considered.

An integrated GNSS system, composed by GPS and GLONASS, is characterized by a significantly increased satellite availability respect to GPS or GLONASS only, ensuring a positioning improvement in "hostile" environments. The performance of the integrated system is increased in terms of:

- Continuity, directly related to satellite availability,
- Accuracy, enhanced by observation geometry improvement and
- Integrity, because the increased availability improves the detection process of gross errors in the measurements set (Angrisano et al. 2010).

The considered GNSS are very similar but with a significant difference in time scales; therefore their combined use involves the addition of a further unknown to estimate, i.e. the intersystem time scale offset, which requires the "sacrifice" of one measurement. A possible way to fully use the GPS/GLONASS combination is the employment of a pseudo-measurement, which takes into account the quasi-constancy of this parameter (Cai and Gao 2009).
A purpose of this research is the performance assessment in difficult scenario of different single point GNSS configurations, with specific interest to investigate the benefits of GLONASS inclusion relative to GPS stand-alone.
Different methods can be adopted to estimate the navigation parameters in single point positioning (using pseudorange and Doppler observables); the most common estimators are Least Squares (LS) (Mikhail 1976, Wells and Krakiwsky 1971) and Kalman Filter (KF) (Kalman 1960, Brown and Hwang 1997).
The LS method is not able to provide navigation unknowns in case of measurement deficiency, while KF guarantee a continuous solution owing to the process model containing equations representing the navigation parameter behavior.

2. GNSS OVERVIEW. GNSS are worldwide, all-weather navigation systems able to provide tridimensional position, velocity and time synchronization to UTC scale (Hoffmann-Wellenhof et al 1992, Kaplan and Hegarty 2006). GPS and GLONASS are herein the considered GNSS, they are similar for many aspects, such as the operational principle described in the next section, but with some meaningful differences detailed in section 2.2.
2.1. *Operational Principle*. GNSS positioning is based on the one-way ranging technique: the time of travel of a signal transmitted by satellites is measured and scaled by speed of light to obtain the satellite-user distance, used to compute receiver coordinates. The receiver clock offset relative to system time scale must be estimated too. The measured range between receiver and satellite is called pseudorange (PR), whose equation is:

$$\rho = d + c\delta t_u + \varepsilon_\rho \tag{1}$$

where ρ is the PR measurement, d is the geometric distance receiver – satellite, $c\delta t_u$ is the receiver clock offset and ε_ρ contains the residual errors after satellite-based and atmospheric error corrections.
Equation (1) holds for both single GNSS (i.e. GPS or GLONASS only) and $c\delta t_u$ is referred to the time scale of the considered system. In multi-constellation case a further unknown, representing the inter-system time offset, must be estimated.
GNSS receivers are also able to provide Doppler measurements, defined as the time derivative of observable phase (Hoffmann-Wellenhof et al 1992, Kaplan and Hegarty 2006) and related to the relative motion between receiver and satellites. Doppler observable is directly converted in a pseudorange rate information and its measurement equation is formally similar to (1) (Kaplan and Hegarty 2006):

$$\dot{\rho} = \dot{d} + c\dot{\delta t}_u + \varepsilon_{\dot{\rho}} \tag{2}$$

where $\dot{\rho}$ is the PR rate measurement, $\dot{d}$ is the time derivative of the geometric distance receiver – satellite, $c\dot{\delta}t_u$ is the receiver clock drift and $\varepsilon_{\dot{\rho}}$ contains the residual errors after satellite-based corrections.

2.2. *GPS-GLONASS Differences*. GPS and GLONASS are very similar but with some meaningful differences, classifiable as: constellation, signal and reference differences summarized in Table 1 (and a detailed in Cai 2009, Angrisano 2010).

Table 1. GPS and GLONASS Comparison (adapted by Cai 2009)

	Parameter	GPS	GLONASS
Constellation	Number of SV	24 (Expandable)	24
	Orbital Planes	6	3
	Orbital Altitude (Km)	20200	19100
	Orbit Inclination (deg)	55°	64.8°
	Ground Track Period	1 Sidereal Day	8 Sidereal Days
	Layout	Asymmetric	Symmetric
Signal	Carrier Frequencies (MHz)	1575.42 1227.60	1602+K*0.5625 1246+K*0.4375
	Ranging Code Frequencies (MHz)	C/A: 1.023 L2C: 1.023 P: 10.23 M: 10.23	C/A: 0.511 P: 5.11
	Multiple Access Schemes	CDMA	FDMA
	Broadcast Ephemerides	Keplerian	ECEF
Reference	Datum	WGS84	PZ90.02
	Time Scale	GPS Time	GLONASS Time

About the constellations, the nominal number of satellites is 24, but GPS constellation provides for the eventuality of surplus satellites with no pre-defined slots. GLONASS orbits are lower than GPS ones and are more inclined, allowing a better coverage at high latitudes. GLONASS satellites orbital period is shorter than GPS one, with ground tracks repeating every 8 sidereal days for the first and every day for the second. Moreover GLONASS constellation has a "symmetric" configuration, i.e. the slots are evenly spaced on each plane and the argument of latitude displacement between the planes is constant (the GLONASS constellation is a Walker constellation). On the other hand GPS constellation is intentionally "asymmetric": the number of satellites on the planes can be different owing to the surplus satellites and the space vehicles are unevenly distributed on the orbit, in order to optimize the constellation coverage in case of one satellite outage (Parkinson and Spilker 1996).

About the signal, all the GPS satellites broadcast signals at the same carrier frequencies L1 and L2, while each GLONASS satellite uses a different carrier frequency. So GPS and GLONASS system use different multiple access schemes: respectively CDMA (the transmitting satellites are distinguished by the code) and FDMA (the transmitting satellites are distinguished by the frequency). The next generation of GLONASS satellites (Glonass-K) is planned to implement the CDMA strategy to improve the compatibility with GPS (Cai 2009).

In addition the chip rate of the C/A and P codes of GLONASS is about half of the corresponding GPS codes. The chip width, defined as the inverse of the chip rate, is related with the receiver high-frequency error. For typical receivers, the standard deviation of this error is about 1/100 of the chip width, corresponding to about 3 m and 0.3 m for GPS C/A and P codes, and to about 6 m and 0.6 m for GLONASS C/A and P codes (Parkinson and Spilker 1996).

Moreover the satellite broadcast ephemerides, stored in the GPS navigation message, are Keplerian parameters and are transformed in Earth Centered Earth

Fixed (ECEF) frame using the orbital propagation algorithm (IS-GPS-200 2004); the broadcast ephemerides in GLONASS navigation message are directly expressed in ECEF frame (ICD-GLONASS 2008), but anyway a propagation algorithm is necessary to compute the satellite position in the desired epoch (usually the epoch of transmission of the signal).

GPS and GLONASS systems adopt different coordinate frames to express the satellite and user coordinates, respectively WGS84 and PZ90, whose details are in IS-GPS-200 2004 and ICD-GLONASS 2008. The two reference frames are nearly coincident, but the measurements combination from both systems require a seven-parameters transformation; neglecting this transformation yields a position error from a single receiver of metric order (Misra et al. 1998). Starting from September 20 2007, an improved version of the GLONASS reference frame is in use, called PZ90.02 (Revnivykh 2007).

GPS and GLONASS systems adopt different reference time scales, connected with different UTC realizations.

In detail GPS time is connected with UTC(USNO), the UTC maintained by US Naval Observatory; UTC scale is occasionally adjusted of one second to keep it close to the mean solar time (connected to the astronomical definition of time). GPS time scale is indeed continuous and so GPS time scale and UTC(USNO) differ for an integer number of seconds (called leap seconds, currently 15). Moreover GPS time and UTC(USNO) are maintained by different master clocks, producing a further difference of typically less than 100 ns; this difference is broadcast to the users in the navigation message.

GLONASS time scale is connected to UTC(RU), the UTC as maintained by Russia. GLONASS time is adjusted by leap seconds, according to the UTC adjustments, so they do not differ for an integer number of seconds, but only for a difference less than 1 millisecond, broadcasted in the GLONASS navigation message.

The transformation between GPS and GLONASS times is expressed by the following formula (Cai and Gao 2009):

$$t_{GPS} = t_{GLO} + \tau_r + \tau_u + \tau_g \tag{3}$$

where $\tau_r = t_{UTC(RU)} - t_{GLO}$ is broadcasted in the GLONASS navigation message, $\tau_r = t_{UTC(USNO)} - t_{UTC(RU)}$ must be estimated and $\tau_g = t_{GPS} - t_{UTC(USNO)}$ is broadcasted in the GPS navigation message.

To perform the transformation (3), the difference between UTC(USNO) and UTC(RU) should be known, but this information is not provided in real-time. This problem is generally solved including the difference between the systems time scales as unknown when GPS and GLONASS measurements are used together.

The GPS-GLONASS system time offset is broadcast via the navigation data as non-immediate parameter included in the GLONASS almanac (ICD-GLONASS 2008), but does not take into account the inter-system hardware delay bias which is dependent on specific receiver (Cai and Gao 2009).

3. ESTIMATION TECHNIQUES.

Estimation is the process of obtaining a set of unknowns (state vector or simply state) from a uncertain measurements set, according to a definite optimization criterion (Bar-Shalom et al. 2001). To estimate the state, a functional relationship has to be defined with the measurements, usually referred to as the measurement model. The discrete and linear version of measurement model is show below:

$$\underline{z}_k = H_k \cdot \underline{x}_k + \underline{\eta}_k \tag{4}$$

with $\underline{z}_k$ measurement vector, H_k design matrix, $\underline{x}_k$ state vector, $\underline{\eta}_k$ measurement noise vector and the subscript k representing the epoch.
The measurement model could be solved for the unknowns if the number of (independent) equations is at least equal to the number of the unknowns. If other equations are included in addition to the measurement model, the set of unknowns can be estimated even in case of measurement lack . These further equations can be obtained considering information about the system state dynamics, usually referred to as process model. The discrete and linear version of process model is show below:

$$\underline{x}_{k+1} = \Phi_{k+1,k} \cdot \underline{x}_k + \underline{w}_k \tag{5}$$

where $\Phi_{k+1,k}$ is the transition matrix and $\underline{w}_k$ is the process noise vector, which take into account the model uncertainty.
The inclusion of the process model can provide in general a better estimation of the system state vector, if the model represents properly the state behavior. The estimation methods adopted in this research are the Least Squares method, using only the knowledge of the measurement and the Kalman filter using also the process model.
3.1. *Least Squares*. The Least Square method is the most common estimation procedure in geomatics application and its estimation process is based purely on the measurements. The LS approach is to obtain a state estimate minimizing the sum of the square residuals, defined as:

$$\underline{r}_k = \underline{z}_k - H_k \cdot \underline{\hat{x}}_k \tag{6}$$

LS solution and the associated covariance matrix are:

$$\begin{aligned} \underline{\hat{x}}_k &= \left(H_k^T W H_k\right)^{-1} H_k^T W \underline{z}_k \\ C_x &= \left(H_k^T W H_k\right)^{-1} = \left(H_k^T R^{-1} H_k\right)^{-1} \end{aligned} \tag{7}$$

The weighting matrix W can be set as the inverse of the measurement covariance matrix R, weighting the accurate measurements more and the noisy ones less (Brogan 1981).
3.2. *Kalman Filter*. The Kalman Filter estimation is a technique commonly used in navigational applications, which uses knowledge about measurements and state vector dynamics and so adopts both measurement (4) and process models (5). The measurement model is formally identical to the model used in LS, with the additional assumption of zero-mean white noise with Gaussian distribution for the measurement noise. The KF is a recursive algorithm using a series of prediction and update steps to obtain an optimal state vector estimate in a minimum variance sense (Kalman 1960, Brown and Hwang 1997).
The prediction step, used to predict the state vector and the associated covariance matrix from the current to the next epoch, is based on the assumed process model:

$$\begin{aligned} \underline{\hat{x}}_{k+1}^- &= \Phi_{k+1,k} \underline{\hat{x}}_k^+ \\ P_{k+1}^- &= \Phi_{k+1,k} P_k^+ \Phi_{k+1,k}^T + Q_k \end{aligned} \tag{8}$$

where the superscript "-" indicates a predicted (or a priori) quantity (i.e. before the measurement update) and the superscript "+" indicates a corrected (or a posteriori) quantity (i.e. after the measurement update). P is the covariance matrix of the state vector and Q is the covariance matrix of the process noise.

The update step is used to correct the predicted state and covariance matrix with the measurements, as shown below:

$$\begin{aligned} \underline{\hat{x}}_{k+1}^{+} &= \underline{\hat{x}}_{k+1}^{-} + K_{k+1}\underline{\nu}_{k+1} \\ P_{k+1}^{+} &= \left(I - K_{k+1}H_{k+1}\right)P_{k+1}^{-} \end{aligned} \tag{9}$$

where K is the Kalman gain matrix and $\underline{\nu}$ is the innovation vector respectively defined as

$$\begin{aligned} K_{k+1} &= P_{k+1}^{-}H_{k+1}^{T}\left(H_{k+1}P_{k+1}^{-}H_{k+1}^{T} + R_{k+1}\right)^{-1} \\ \underline{\nu}_{k+1} &= \underline{z}_{k+1} - \underline{\hat{z}}_{k+1} = \underline{z}_{k+1} - H_{k+1}\underline{\hat{x}}_{k+1}^{-} \end{aligned} \tag{10}$$

The innovation vector can be considered as an indication of the amount of information introduced in the system by the current measurements. The Kalman gain matrix is a weighting factor, indicating how much the new information contained in the innovation vector influences the final state vector estimate.

4. IMPLEMENTATION.

4.1. *PVT Algorithm*. In this research PVT (Position-Velocity-Time) algorithms (detailed in figure 1) are developed in Matlab environment to process GNSS data in single point mode; the software belongs to a tool implemented at PArthenope Navigation Group (PANG).

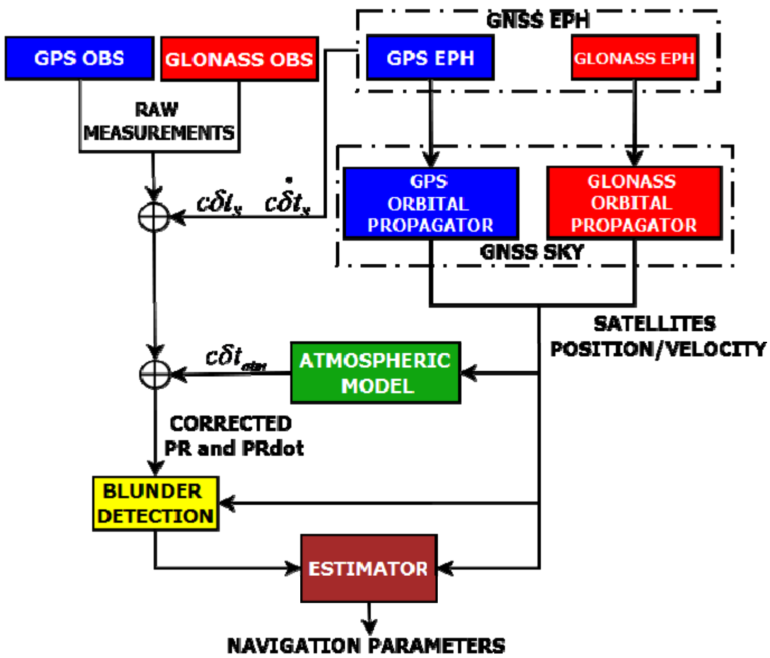

Figure 1. PVT Algorithm Scheme

Main inputs are the GNSS raw measurements, i.e. pseudorange and Doppler, and the GNSS ephemerides.

The ephemerides are used to compute satellite position and velocity; different orbital propagators are implemented for the considered GNSS because the ephemerides are differently parameterized. The GPS orbital propagator is

extensively treated in IS-GPS-200 2004 and Remondi 2004, while for GLONASS the main reference is ICD-GLONASS 2008.
Measurements are corrected for satellite clock and atmospheric errors, specifically Klobuchar and Hopfield models are adopted to reduce ionosphere and troposphere delays respectively.
A quality check is performed epoch by epoch on the corrected measurements to detect and reject gross errors; the strategy adopted is the "observation subset testing" (Kuusniemi 2005), using the global test as a decision parameter. The quality control is performed testing the residuals in the LS case and the innovation vector in KF case; the measurement errors are assumed to be Gaussian with zero-mean and uncorrelated. The decision variable is defined as the sum of the squares of the residuals (or innovations), weighted by the measurement covariance matrix:

$$D_{LS} = \underline{r}^T \cdot R \cdot \underline{r} \quad ; \quad D_{KF} = \underline{v}^T \cdot R \cdot \underline{v} \tag{11}$$

and is assumed to follow a χ^2 distribution with $(m-n)$ degrees of freedom or redundancy, defined as the difference between the number of measurements and states.
The threshold T is usually related to probability of false alarm and redundancy as shown below:

$$T = \chi^2_{1-P_{FA},(m-n)} \tag{12}$$

being the abscissa related to a probability value $(1-P_{FA})$ of a chi-square distribution of $(m-n)$ order.
A common procedure consists of fixing P_{FA} according to the application requirements and letting the threshold vary with the redundancy; a typical value for the probability of false alarm is 0.1% (Petovello 2003).
The described procedure (global test) is applied to the whole set of measurements: if it passes the test, the measurements are considered self-consistent and no rejection is carried out, otherwise the procedure is applied to all the possible subsets including measurements from $(m-1)$ to $(n+1)$ in order to identify a subset passing the global test (Kuusniemi 2005).
The aforesaid blunder detection technique is applied separately to pseudorange and Doppler observations.
After the blunder rejection, the measurements are processed with LS and KF methods. The measurement model consists of equations as (1) and (2), linearized for the unknowns, and assumes the following expression:

$$\underline{\Delta\rho} = H \cdot \underline{\Delta x} + \underline{\varepsilon} \tag{13}$$

where $\underline{\Delta\rho}$ is the difference between actual and predicted measurements, $\underline{\varepsilon}$ is the residual error vector, $\underline{\Delta x}$ is the state vector, detailed below

$$\underline{\Delta x} = \left[\underline{\Delta P} \quad \underline{\Delta V} \quad \Delta\left(c\delta t_u^{GPS}\right) \quad \Delta\left(c\dot{\delta} t_u^{GPS}\right) \quad \Delta\left(c\delta t_{sys}\right) \right]^T \tag{14}$$

The state vector contains the receiver position, velocity and clock errors used to correct the previous navigation parameter estimation. $c\delta t_{sys}$ is the difference between GPS and GLONASS time scales.

A constant velocity model is adopted for the process, with velocity errors being modeled as a random walk process and $c\delta t_{sys}$ as a random constant process to take into account its quasi-constancy (Cai and Gao 2009).

Developed PVT algorithms operate in a closed-loop mode, i.e. every epoch the state vector is estimated and is used to correct the nominal state, then the state vector is reset to a null vector (Brown and Hwang 1997, Godha 2006). The strategy is preferred to open-loop, because errors on the estimated navigation parameters are small enough to maintain valid the assumptions for the linearization process.

4.2. *Aiding on Inter-System Time Scale*. If GPS and GLONASS measurements are used together, the difference between the systems time scales must be estimated, limiting a full utilization of the multi-constellation, because one equation is "sacrificed" to estimate the further unknown.

The offset between GPS and GLONASS time scales can be considered constant in a brief interval (Cai and Gao 2009), hence a pseudo-measurement, observing directly $c\delta t_{sys}$, can be introduced as follow:

$$\left(c\delta t_{\text{sys-AID}} - c\delta t_{\text{sys0}}\right) = \left[(0)_{1x8} \quad 1\right] \cdot \underline{\Delta x} \tag{15}$$

Equation (15) can be included in the measurement model (13), allowing a GPS/GLONASS solution in LS case with 4 mixed visible satellites; this aiding is also used in case of sufficient measurements (≥5 mixed satellites) to enhance measurement model redundancy.

5. TEST.

5.1. *Description*. The data collection is a vehicular test and was carried out on 22nd July 2010 in the afternoon in downtown Calgary (Canada), typical example of urban canyon; many GNSS signals are blocked by skyscrapers or are strongly degraded for the multipath problems. The test begins in a small parking lot with a static period in good visibility condition (9 GPS and 5 GLONASS available satellites) and continues into the downtown core where the number of visible satellites decreases significantly, bringing to many partial and total GNSS outages during the trajectory (Figure 3). The test finishes outside downtown with good visibility conditions. The total duration of the test is about 30 minutes, the vehicle speed varies from 0 to 50 km/h with frequent stops due to the traffic lights and the total distance travelled is about 10 km. The trajectory followed by the car is shown in Figure 2.

Figure 2. Test Trajectory

Figure 3. Urban Segment

5.2. *Equipment*. The receiver used, a NovAtel ProPak-V3 belonging to the OEMV family, is a high-performance device able to provide L1 and L2 GPS+GLONASS positioning; the connected antenna is a high performance NovAtel 702 antenna and is mounted on the car roof as showed in the Figure 4.


Figure 4. Equipment

5.3. *Reference*. The device used for generating a reference solution is the NovAtel SPAN (Synchronous Position, Attitude and Navigation) system, consisting of a Honeywell HG1700, a tactical grade IMU (Inertial Measurement Unit), and an OEM4 GPS receiver. The NovAtel ProPak-V3 and OEM4 receiver are connected to the same antenna through a signal splitter. The reference solution is computed in post-mission, processing the inertial and the GPS data with the NovAtel Inertial Explorer software, using the tightly coupled strategy and the double difference technique; the GPS base station for differential processing is placed on the roof top of a building 6-7 km away from the test location. The reference solution accuracy in these conditions (as estimated by the NovAtel software) is decimetric for the position and cm/s for the velocity.

6. RESULTS AND ANALYSIS. In this research 4 GNSS configurations are considered and analyzed, differing for satellite system and estimation method, specifically:

- GPS only with LS (GPS LS),
- GPS/GLONASS with LS (GG LS),
- GPS only with KF (GPS KF),
- GPS/GLONASS with KF (GG KF).

Pseudorange and Doppler observations are processed in single point positioning. The comparison is carried out in terms of solution availability (i.e. the percentage of time mission when solution is available) and position/velocity accuracy; for a fair comparison, accuracy analysis is performed when the solution is obtainable for all configurations (i.e. if GPS LS fix is available).

Table 2. Solution Availability

Solution Availability			
GPS LS	**GG LS**	**GPS KF**	**GG KF**
0.61	**0.65**	**1**	**1**

KF solutions are continuous, hence solution availability is 100%; GPS LS solution is characterized by several partial and total outages (clearly visible in figure 5 on the top) and the fix is possible during only 61% of the mission (table 2). The GLONASS inclusion bring to a 4% of improvements in availability (table 2) and to a reduction of GNSS outages (circled areas in figure 5).

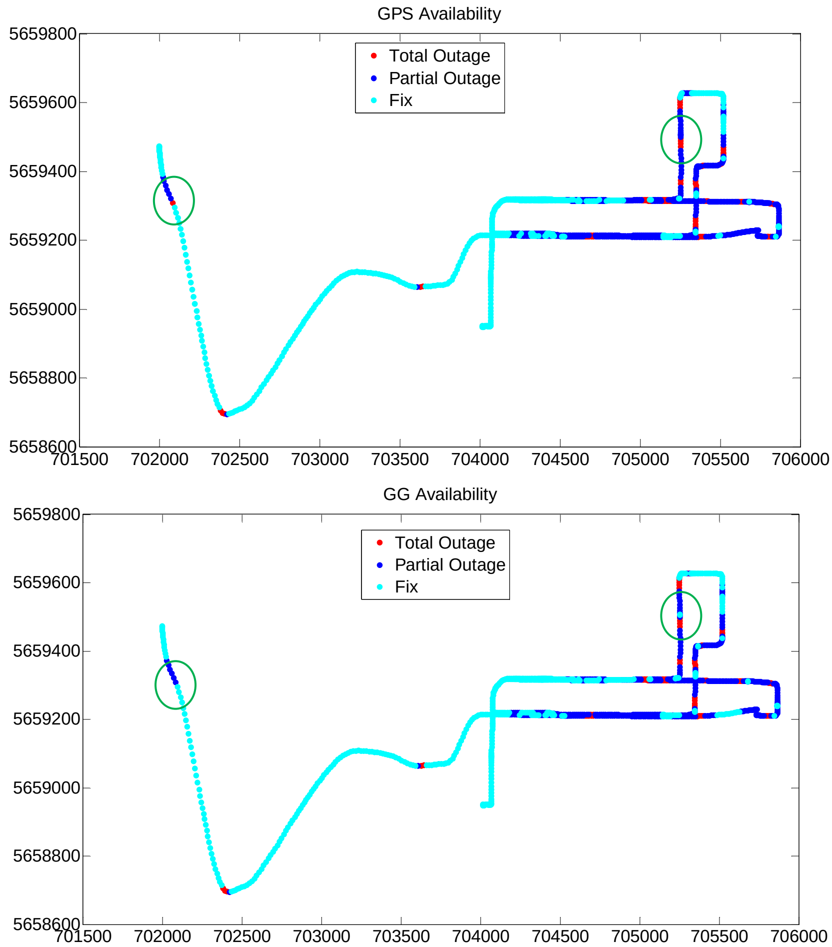

Figure 5. LS Solution Availability on the Trajectory

The accuracy analysis is carried out in terms of RMS (Root Mean Square) and maximum errors on position and velocity and is summarized in tables 3 and 4.

Table 3. Position Accuracy

Configurations	RMS (m)			Max (m)		
	Horizontal	*Up*	*3D*	*Horizontal*	*Up*	*3D*
GPS LS	**11.4**	**20.6**	**23.5**	**196.3**	**210.9**	**278.2**
GG LS	**9.4**	**15.1**	**17.8**	**97.0**	**204.7**	**223.5**
GPS KF	**10.2**	**19.0**	**21.6**	**51.5**	**179.8**	**184.4**
GG KF	**9.4**	**7.5**	**12.0**	**54.8**	**41.1**	**61.8**

The GPS/GLONASS configurations demonstrate improved performance with respect to GPS only (line 1 versus line 2 and line 3 versus line 4 in table 3) in both horizontal and vertical components, in terms of RMS and maximum error.
RMS horizontal errors are similar for homologous LS and KF configurations, but KF limits maximum horizontal errors. Moreover KF vertical errors are strongly limited too because the process model is consistent with altitude slow variations in typical vehicular navigation.

Table 4. Velocity Accuracy

Configurations	RMS (m/s)			Max (m/s)		
	Horizontal	*Up*	*3D*	*Horizontal*	*Up*	*3D*
GPS LS	**0.188**	**0.254**	**0.316**	**2.982**	**3.491**	**4.591**
GG LS	**0.170**	**0.224**	**0.281**	**2.803**	**3.045**	**4.138**
GPS KF	**0.180**	**0.225**	**0.288**	**2.812**	**3.028**	**4.133**
GG KF	**0.168**	**0.202**	**0.263**	**2.695**	**2.674**	**3.796**

From table 4 it can be noted that velocity solutions are very similar, with only slight advantages for GPS/GLONASS on GPS only and KF on LS.
The multi-constellation approach provides improvements, in terms of solution availability and accuracy, but it is necessary to introduce an additional unknown, hence one observation is used to estimate it. The time scale offset between GPS and GLONASS time can be considered constant in a brief interval, hence a pseudo-observation is used to observe directly the unknown. The pseudo-observation is based on "old" estimation of $c\delta t_{sys}$ in good accuracy condition, i.e. with low value of the corresponding element of solution variance/covariance matrix.
The configuration with the aiding (referred to as GG LS Aiding) is compared with standard GG LS solution, showing a solution availability improvement of 3%, owing to the fix performed in case of 4 mixed satellites (as shown in table 5).

Table 5. Solution Availability with Aiding

Solution Availability	
GG LS	**GG LS Aiding**
0.65	**0.68**

The aiding is used always (not only in case of 4 mixed GPS/GLONASS) as further measurement to increase the redundancy and shows improved performance with respect to GG LS case, in both horizontal and vertical components, in terms of RMS and maximum error.

Table 6. Position Accuracy with Aiding

Configurations	RMS (m)			Max (m)		
	Horizontal	Up	3D	Horizontal	Up	3D
GG LS	9.4	15.1	17.8	97.0	204.7	223.5
GG LS Aiding	8.0	13.4	15.6	64.1	204.7	210.9

The inclusion of the aiding on $c\delta t_{sys}$ does not produce benefits on GG KF configuration, because the quasi-constancy of the inter-system time scale bias is just included in the process model.

7. CONCLUSIONS. Based on the research results presented in this paper, GPS/GLONASS configurations show evident improvements with respect to GPS only in terms of solution availability and accuracy, usually considered critical parameters in urban scenario. Least squares and Kalman Filter estimators are used to process GNSS data in single point positioning, for both methods GLONASS inclusion yields evident benefits.
The multi-constellation systems bring to the estimation a further unknown, i.e. the offset between their time scales; to avoid the “sacrifice” of one observation, a pseudo-measurement, observing directly the offset, is introduced taking into account its stability for short periods. With the aiding on the inter-system timescale offset, the required minimum satellite number to LS solution estimate is reduced to four, instead of the five required by the standard model; the GG LS aided solution demonstrates improved availability and accuracy, while no benefits are noticed in GG KF case (offset quasi-constancy just included in the process model).
The estimation methods considered provide similar performance in terms of RMS, but KF solutions demonstrate better performance with respect to LS homologous configurations in terms of maximum errors and in the vertical solution; this can be explained considering that the simple process model adopted well represents the slowly varying altitude behavior and is able only to limit great errors in the horizontal solution but is not consistent with the actual vehicle motion.

8. FUTURE WORK. The result obtained demonstrate the benefit of the GLONASS inclusion, the next step is the performance assessment of a multi-constellation system including Galileo.
Further pseudo-measurement will be introduced in the measurement model; starting from the encouraging results of this study the authors at first will investigate the performance of the aiding on the altitude and the combined use of altitude and $c\delta t_{sys}$ pseudo-measurements.
It will be also included in the next step of the research, the development of an adaptive KF for vehicular navigation.

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